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Technological advancement, learning, and the adoption of new technology

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Abstract

This research focuses attention upon three related issues: the persistence of technological progress over time, multiple adoption decisions over a long horizon, and the impact of production based learning on those decisions. We develop a simple economic model for a single product producing firm incorporating continuous technological progress, linear product demand, linear switching costs, and experience based cost reductions. A dynamic programming framework is used to evaluate cases where either one or an unlimited number of adoptions are allowed over an infinite horizon. Both structural and numerical results are presented. Fundamental results serve to explain several counter-intuitive dynamics. In the single adoption case, faster rates of technological progress, as well as growing markets, or a steeply sloping demand curve, serve to delay adoption while an increased ability to learn may accelerate it. When multiple adoptions are allowed, the adoption of any particular technology presents a "window of opportunity" in which future investment will be warranted. If, for whatever reason, this window has passed, then maintaining the older technology becomes more attractive than adoption. Thus, seemingly outdated technologies may remain embedded in some settings.

Keywords: Replacement; Manufacturing; Dynamic programming; Technology management

1. Introduction

The production competence of a firm has dramatic implications for the firm's strategy, competitive strength, and financial performance. While production competence is affected by a wide variety of factors, it is clear that one of the most significant is the stock of technological assets. Insights guiding the adoption decisions for assets which embody technological improvements are therefore of utmost importance. These decisions have major and immediate impact on the bottom line for at least two reasons. First, they often involve a substantial up front investment of capital. The cost of "switching" from an older technology to a new one includes the costs of changes in, machinery, tooling, training, and production methods. Second,

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adoption decisions largely determine production costs. Improved technologies are often capable of reducing unit production costs substantially, thus improving profit margins.

In many settings, technological capability is embedded in new machinery or tools that appear as discontinuities or "breakthroughs." This notion of technological progress is often modeled in the OM and Economics literature today. However, this model is not always accurate. In some settings technological progress may be better modeled as continuous improvement. For example, an industry or production process may be very stable in terms of the methods used and the technology applied. More commonly, the relevant technology is a complex amalgam of components each of which improves incrementally over time. When these components are viewed as a system, the pattern of performance improvement may appear less as discrete "jumps" and more as a continuous process.

The evolution of computer capabilities is often modeled as a series of jumps or steps. Consider what \$2600 to \$2800 bought in a PC in 1986, 1991, and 1996. Dell first offered an "AT-class" computer in January 1986. \$2852 purchased a "6 MHz 286 machine with 640 K DRAM (no cache), one diskette, a 20 MB hard disk, a 12-inch monochrome monitor", and no software. In their August 1991 ad, Gateway 2000 asked \$2795 for a "33 MHz 386 with 4 MB DRAM, 64 K cache, two high density diskette drives, a 200 MB hard disk, a 14 inch SVGA color display, a 16-bit display adapter with 1 MB of display RAM, a mouse, DOS, and Windows 3.0". The August 1996 Gateway 2000 ad showed the following system for \$2699: "166 MHz Pentium, 32 MB synchronous RAM, 256 K pipelined burst cache, one high density diskette drive, a 2.5 GB hard disk, a 17-inch display, a 128 bit local bus display adapter with 2 MB of RAM, a mouse, Windows 95 and an 8 speed CD-ROM drive with Microsoft Office 95 Professional" (Woods and Wilton 1997).

It is incomplete to focus simply upon generations of processing chips. Therefore, it is not entirely accurate to model this progress as "jumps" from the 1986 system to the 1991 system to the 1996 system. In this setting major technological progress is made by a host of parties. Developers such as Intel push the development of CPU and internal control chips. Software developers, including Microsoft, constantly revise and introduce new products to better leverage established machine capabilities or to take advantage of emerging ones. A host of companies serve to improve other elements of the system such as displays, disk drives, printers, interfaces, keyboards, power supplies, backup systems, etc. With so many components and so many incremental improvements to each one, it becomes impossible to focus on singular jumps to satisfactorily describe technological progress. As a result, the capabilities of comparably priced systems appear to increase over time in a dramatic but virtually continuous fashion.

Within this environment, many companies place more structured constraints on the adoption process. Budget cycles for capital expenditures often create decision points where "lumps" of capital are allocated for technology acquisition. These decisions are often made after a periodic review process that may take place once per year or once per quarter. This research seeks to model the interaction between continuous technological progress and periodic technology adoption decisions.

1.1. Previous research

Extensive reviews of the existing technology adoption literature have been written by Fine (1993) and Bridges et al. (1991). It is useful to divide this literature based upon whether it is focused on specific technologies or views technology in a more general way. For example, the emergence of Computer Aided Design, Computer-Integrated Manufacturing, and Flexible Manufacturing Systems has motivated a body of work on their economic valuation. (See Fine, 1993; Buzacott and Yao, 1987, for detailed reviews of this literature.) On the other hand, a large body of literature takes a broader view of technology management. Among these, many researchers focus upon firms making periodic decisions about technology adoption.

To a great extent, these technology adoption models have evolved out of the equipment replacement literature. The general structure of these settings is that the firm holds a machine whose performance is the "state of the art" upon purchase. However, over time an "inferiority gap" emerges between the firm's machine and the new state of the art. Most of the early equipment replacement models assumed that this gap grows due to deterioration of the existing equipment. (See Pierskalla and Voelker (1976) for a more extended review.) Derman (1963) models stochastic deterioration of existing equipment that can be replaced by new equipment of the same type. The general solution for these types of models is for the firm to adopt the newest technology every time that the gap crosses some threshold. This "inferiority gap" may grow deterministically or stochastically depending on the assumed form of deterioration.

In recent years the pace of technological progress appears to have increased. In this environment, existing equipment is often replaced not because it is worn out, but because its capabilities have been superceded by improved models or methods. For example, Hopp and Nair (1994) extend Derman's model by introducing a single improvement in technology arriving at an uncertain future point. The fundamental result of these efforts has been to show that while a control limit structure is maintained, its nature becomes much more complex when technological progress is included.

The stochastic dynamic programming model of Balcer and Lippman (1984) has been highly influential in this area. In that research, a single firm has some "technology level" labeled "x", while the "state of the art" in manufacturing technology is defined as level "y". A parameter "i" is defined as an index of "discovery potential." This value is actually a parameter in the distribution functions that describe both the timing of the next breakthrough and the magnitude of its impact on production cost. The fundamental result is consistent with earlier models in that the decision to invest is based on a cutoff point or "technology gap" ($y - x$). The model further implies that if the pace of technological progress increases (larger values of i) then the value of $y - x$ that motivates technology adoption also increases, possibly causing a slower rate of adoption. [1]

Cohen and Halperin (1986) also model the adoption decision using a dynamic, stochastic model where the impact of technological advancement is to present opportunities to acquire assets with higher fixed costs but reduced variable costs. Their model also produces optimal production plans in the process of evaluating new technologies. Grenadier and Weiss (1997) apply an options pricing approach to focus upon when a firm should adopt the newest technology as opposed to waiting until the next generation arrives. At that point, the firm has the option to "leapfrog" a generation of technology, or to acquire an older generation at a reduced cost. Farzin et al. (1998) also apply an options pricing approach to uncover the adoption policy for a single firm facing stochastic inter-arrival times for new technologies. This model focuses on innovations that will increase the firm's "productivity," but an increase in productivity is synonymous with a reduction in variable production costs. Chambers and Kouvelis (2000) develop a game theoretic model in a dynamic, stochastic environment, in which duopolists select both output rates, and investment rates in new technologies. Their model is different in that the investment decision focuses on investment levels and does not assume that investment is an "all or nothing" proposition.

All of these works share three characteristics that are not present here. Each of these works assumes that technology progresses by taking steps or jumps such that the performance gap is monotonically increasing. This is not the most realistic description in some cases. Progress may be more continuous, and as costs approach some natural lower bound, the evolution of the performance gap may be more complex. Much of the focus of the work mentioned earlier is on the impact of the uncertain timing of technology introductions. In this research we wish to isolate and explicitly account for the impact of the rate of technological evolution over a long horizon. Finally, it is generally assumed that decision makers can instantaneously adopt new technology at its moment of first availability or whenever a critical threshold is reached. While historically this was a minor assumption, it is of growing importance as the pace of change accelerates.

[1] Balcer and Lippman actually make the stronger claim that under some rather general assumptions increasing i values necessarily results in decreasing adoption rates. Recent work by Kornish (1999) has proven that this conclusion is not justified, but that increasing levels of i may or may not lead to lower adoption rates, depending on other problem parameters.

1.2. Outline of this research

Here, we focus attention on how the firm's periodic adoption decisions are affected by the evolving market of technological assets and the evolving capability of the firm to utilize these assets. We consider a single firm making a single commodity facing linear demand. Technology is "advancing" at a fixed rate (progress rate ρ) and manufacturing experience is affecting production costs in a predictable way (learning rate α).

This paper's basic assumptions include the definition of "technology" to explicitly mean "the total body of applicable knowledge that can be brought to bear to reduce production costs". [2] It is understood that the development of technology occurs due to the efforts of a large number of agents external to the firm under study, and that a single firm's adoption of, or investment in technology has no effect on the development process. Finally, we assume that experience reduces manufacturing costs in each period and that this experience is not transferable to a new technology.

This explicit focus upon production costs alludes to the types of investment being considered. Our primary consideration here is technologies that produce gains in production efficiency. Examples of technologies that improve efficiency include investments in:

- Machinery that enables farmers to produce a given amount of wheat (at a given level of quality) at lower costs.
- Back-office automation that enables firms to complete traditional administrative functions such as payroll processing, faster and cheaper.
- Prototyping tools, usability labs, and computer-aided design tools that enable firms to design and develop a given product at lower costs, and
- E-commerce tools, such as electronic reservations systems, web-based support tools, and other technologies that enable firms to distribute a given product (or provide a given service) at lower costs (Thatcher and Oliver, 2001).

One major area that this approach does not account for is investment intended to improve product quality. Quality improvements are realized when a technology investment leads to the creation of new products, or new features for existing products, which directly increase human desire to consume those products. Examples of technology investments that improve product quality include investments in:

- Patient tracking systems by hospitals that enable emergency room doctors to provide better care in a more timely manner to those patients in greatest need.
- Data mining tools used by credit card companies and grocery stores to sift through customer data to identify patterns that allow them to develop targeted product offerings.
- Interactive TV technologies and advanced internet services by telecommunications providers that enable them to provide new product features to consumers.
- Decision support systems and group decision support systems that enable individuals and groups to organize data and communicate ideas more effectively, leading to better decision-making capabilities (Thatcher and Oliver, 2001).

Investments to reduce costs and investments to improve quality are both ubiquitous in industry. Further research may extend the focus of this paper to envelop both motivations for investment, but here we chose to restrict ourselves to a focus on costs.

[2] This definition is consistent with that used by Balcer and Lippman (1984).

In Section 2 we develop the basic decision model for a single firm. We consider the setting in which the firm will switch to a new technology only once over its planning horizon. In Section 3 we generalize the analysis to the case where technology adoption is considered and can be done at regular intervals. In both scenarios we seek structural results and provide a numerical example for illustrative purposes. Concluding remarks are offered in Section 4.

2. Single adoption case

Consider a firm manufacturing a single product. This firm chooses output rates "R" and sells its output at the market clearing price. The price per unit obtainable with an output rate of R is simply $(M - \sigma R)$ where σ is the slope of the demand function, and M is an exogenous parameter defining market size. The gross revenue resulting from this production is; $R(M - \sigma R)$.

We will use "t" to index the time periods. At the beginning of the problem horizon, $t = 0$ and the firm has an asset base that will allow it to produce a unit of output with a marginal cost "C." We allow this cost to evolve as the result of learning. Assuming some fixed technology that has been in place for n periods we define the marginal cost per unit in that period as, $c = C\alpha^n$, where $0 \leqslant \alpha \leqslant 1$, is the learning rate. Thus $(1 - \alpha)$ indicates how much the marginal cost drops with a period of experience. Note that $n = t$ before a switch takes place but reverts to 0 once the firm switches to a new technology. With these definitions in place we may state production cost during a single period (before a switch occurs) as, $RC\alpha^n$.

We define the best available production cost as $C\rho^t$, where ρ is a parameter indicating how well the marketplace "learns" to reduce manufacturing costs. Since this environment consists of a large body of technology developers and researchers, the progress of technology should be faster than the pace of internal learning, thus $\rho < \alpha$.

It the firm switches to the new technology in period T, it pays a switching cost. We chose here to use a switching cost function that is linear in the per unit cost savings that it generates. Generally we label the switching cost "K" as defined in (1) below,

$$K = k + s(y - x) \tag{1}$$

where "k" is the fixed portion of the switching cost, "s" is the parameter relating per unit cost savings to the market price for technology, "y" is production cost associated with the currently installed technology and "x" is initial production cost available after adopting the state of the art technology.

At the moment that a switch occurs (time T) we move from one cost, $C\alpha^T$ to a new one $C\rho^T$. Eq. (1) is more explicitly applied to this case in (2) below.

$$K(T) = k + sC(\alpha^T - \rho^T) \tag{2}$$

We may combine gross revenue with the marginal cost to define net revenue earned in period t, "$\Pi(t)$" in Eq. (3).

$$\Pi(t < T) = RM - \sigma R^2 - RC\alpha^n, \quad \Pi(t \geqslant T) = RM - \sigma R^2 - RC\rho^T\alpha^n \tag{3}$$

We can generalize these two cases as,

$$\Pi(t) = RM - \sigma R^2 - RC\rho^z\alpha^n \tag{4}$$

Note that t always equals the period of the latest switch (z) plus the age of the current technology (n), and that when $t < T$, $n = t$ and $z = 0$. After a switch occurs in period T then $z = T$ and $n = t - T$. Since (4) is a concave function of R, we may use first order conditions to determine an the profit maximizing output rate R^* yielding, $R^* = M - C\alpha^n\rho^z/2\sigma$. By substitution this implies that the maximum obtainable profit in a single period is,

$$\Pi^* = \frac{(M - C\alpha^n \rho^z)^2}{4\sigma} \tag{5}$$

2.1. Decision formulation

We introduce a discount rate β such that \$1 received in period $(t+1)$ is worth \$$\beta$ in period t, thus our objective is to chose a switching point T in order to maximize the present value of net profit labeled $\Phi(T)$ as defined in Eq. (6).

$$\max_T \Phi(T) = \sum_{n=0}^{T} \frac{(M - C\alpha^n)^2}{4\sigma}\beta^n + \beta^T \sum_{n=0}^{\infty} \frac{(M - C\rho^T\alpha^n)^2}{4\sigma}\beta^n - K(T)\beta^T \tag{6}$$

Fig. 1 shows the evolution of production costs over time. The solid line is $C\rho^t$ and represents the lowest attainable production cost using the state of the art technology. The dashed line shows production cost before a switch is made ($C\alpha^n$). At time T ($T = 10$ for this illustration), the firm buys the ability to move down to point "A" on the lowest cost curve. From this point forward, the marginal cost decreases due to learning over the remaining horizon ($c = C\rho^T\alpha^n$). The vertical drop lines show the gap between the realized cost and the lowest possible cost. Note that this gap is increasing immediately after times 0 and T but eventually begins to decrease as t becomes larger. Fig. 2 shows Φ values for a typical problem.

Note that Φ may be increasing, decreasing, convex, or concave, depending on parameter values and the period t. However, for this model to make economic sense we must have $0 \leqslant \rho \leqslant \alpha \leqslant 1$, and $\beta \leqslant 1$. We also wish to ignore uninteresting cases where costs are always negligible or markets are too small to be of interest. Thus if we assume that $C > 4\sigma/(1-\alpha^2)\alpha^2$ (Assumption A) and $M > 2C\{\alpha(1-\alpha)^2 + (1-\rho)^2\}/(1-\alpha)(1-\rho^2)$ (Assumption B) we may prove Proposition 1 directly.

Proposition 1. *Under Assumptions A and B, Φ is uni-modal.*

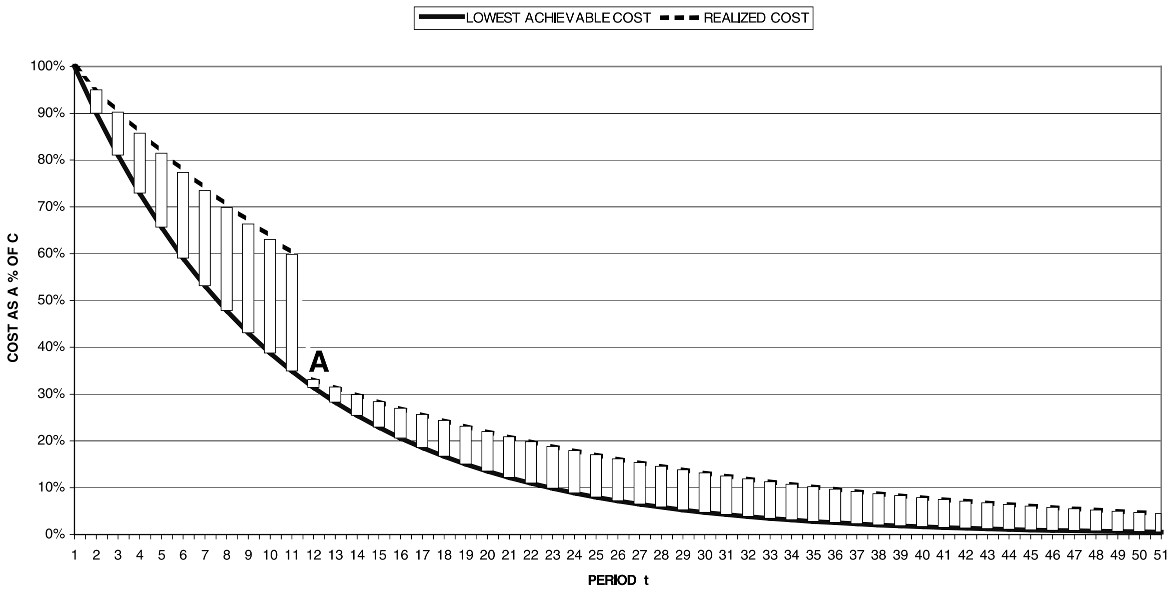

Fig. 1. Cost difference for old vs. new technology.

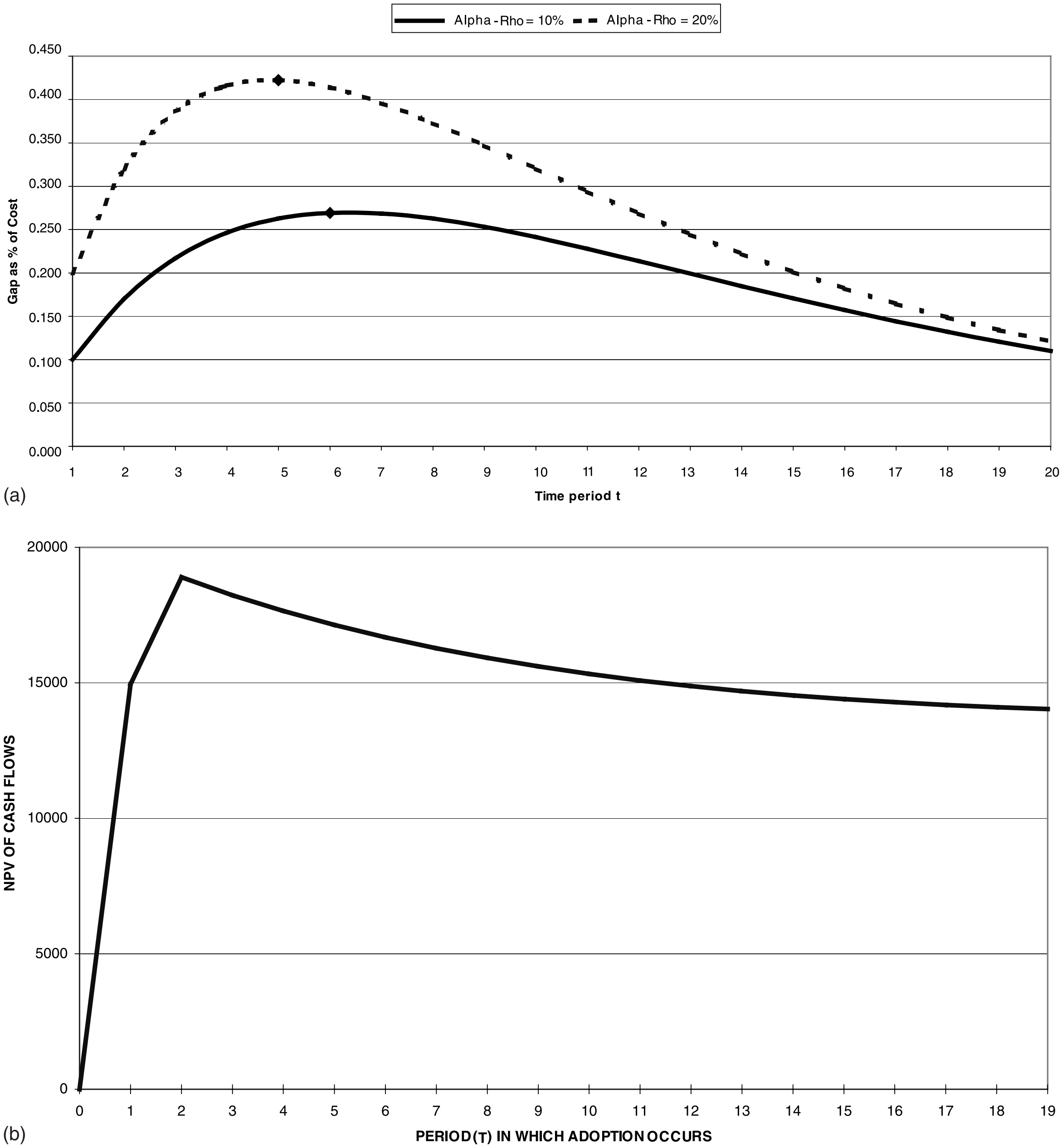

Fig. 2. Total NPV vs. switching point.

In other words if Φ is decreasing in t over a range from t to $t+1$, then it cannot be increasing in t over a range from $(t+1)$ to $(t+2)$. (The proof of each Proposition is offered in the appendix.) Therefore, if $\Phi(1) \geqslant 0$ and $\Phi(1) \geqslant \Phi(2)$ then it is optimal to switch at time $(t=1)$. If $\Phi(2) > \Phi(1)$, then we should compare $\Phi(2)$ to $\Phi(3)$ to determine if switching at time 2 is better than waiting until $t=3$ and so on. If we define $\Psi(t)$ as $\Phi(t) - \Phi(t+1)$ we may state the decision rule formally, switch iff, $\Phi(t)$ and $\Psi(t) > 0$ where,

$$\Psi(t) = A\rho^t + B\alpha^t + D\rho^{2t} + E\alpha^{2t} + (\beta - 1)k \tag{7}$$

and,

$$A = \frac{(\rho\beta - 1)MC}{2\sigma(1 - \alpha\beta)} + (1 - \beta\rho)sC, \quad B = \frac{2MC}{4\sigma} + (\alpha\beta - 1)sC$$

$$D = \frac{(1 - \beta\rho^2)C^2}{4\sigma(1 - \alpha^2\beta)}, \quad E = \frac{C^2}{4\sigma}$$

2.2. Structural results

Eq. (7) may be analyzed to focus upon the drivers of the adoption decision. The switching point is the time T when Ψ first becomes positive, therefore, events which increase Ψ for small t values lead to earlier adoption while those that decrease Ψ serve to delay adoption. Analysis of (7) leads to Proposition 2.

Proposition 2. *T is increasing in k, and M, and decreasing in s. When M is sufficiently large, T is also increasing in β, and decreasing in σ.*

This result is driven by three factors: the concavity of the benefit of switching, the cost of switching, and the effect of discounting. Recall that the gap between the manufacturing cost without switching and the best available cost is $C(\alpha^t - \rho^t)$. The firm has a motivation to delay switching technologies until this difference is maximized. Proposition 2 addresses the extent to which changes in parameter values makes it less attractive to wait until this maximum occurs.

The fixed portion of the switching cost, k is paid whenever a switch occurs. Since discounting makes it more attractive to pay k tomorrow, rather then today, $\Psi(t)$ is decreasing in k. Thus increasing k delays adoption.

A larger market provides a setting in which a cost savings per unit produced creates greater benefits. In a large market, even a small change in production costs can provide a large benefit. Thus the firm is more strongly motivated to wait until $C(\alpha^t - \rho^t)$ is maximized. Recall that the price per unit is $(M - \sigma R)$. Therefore, increasing σ values would have the same effect as decreasing the market size. Consequently T is both increasing in M and decreasing in σ.

The fact that $C(\alpha^t - \rho^t)$ is concave as it approaches its maximum helps to explain the way in which T is effected by changes in s. Only as long as the benefit to switching is growing faster than the cost (which rises at a rate s), is it reasonable to delay adoption. Thus a higher value of s makes it less attractive to wait for a minor increase in cost savings. Since Ψ tends to become flat as it reaches its peak, T is decreasing in s.

In the absence of discounting, any cost savings related in each period over an infinite horizon has an infinite value; thus, we would always delay adoption until this savings is as large as possible. On the other hand, the presence of discounting implies that it is only reasonable to postpone the receipt of a benefit for one period if the increase in that benefit is greater than the decline in its value resulting from delaying its receipt. As β rises, we are more willing to wait until the maximized cost savings is achieved. Thus T must decrease as discounting becomes more significant (β falls). In other words, T falls as β falls and rises as β rises.

Since T is primarily a function of $(\alpha^t - \rho^t)$, it is not always increasing or decreasing with changes in α and ρ. Fig. 2a highlights the effect of increasing the gap between α and ρ on the cost difference that can be gained by switching technologies.

We note that increasing the difference between α and ρ has two effects on $C(\alpha^t - \rho^t)$. First we see that the maximum change in cost that may be achieved by technology adoption is greater when $\alpha - \rho$ is greater.

Table 1
Profit and switching cost considering a single adoption

Period T	Profit before switch	Profit after switch	Cost to switch	Total profit Φ (T)
1	681	14,355	90	14,945
2	1398	17,605	109	18,893
3	2134	16,210	116	18,228
4	2874	14,888	114	17,649
5	3607	13,636	107	17,136

Second, we also see that the point in time at which this peak occurs is earlier when $\alpha - \rho$ is greater. Consequently, decreasing the gap between α and ρ has two distinct effects. It postpones the curve's peak, serving to delay adoption. However, it also flattens the curve implying that the decision maker may not be as willing to wait until this peak is reached. Consequently, whether increasing the gap between α and ρ increases or decreases T depends on the other parameter values.

2.3. An illustrative example

Consider the following parameter values. The market is characterized by $M = 100$, and $\sigma = 1$. The product in question has an initial production cost, $C = 50$. The switching cost is characterized by $k = 50$ and $s = 10$. We assume learning and progress rates of $\alpha = 90\%$, and $\rho = 80\%$ and a discount rate $\beta = 90\%$.

Table 1 focuses upon the components of Eq. (6) and shows results obtained by setting T to values ranging from one through 5. The first column shows T, the number of periods that the firm employs the older technology before switching. The second column shows the present value (at time $t = 0$) of all net revenue incurred before the switch occurs. Column 3 shows the present value of the revenue earned after the switch occurs. The present value of the cost to switch is shown in column 4, and the final column shows the total profits minus the switching cost.

As the table shows, it is optimal to switch in period 2. Note that the gap between $C\alpha^t$ and $C\rho^t$ actually peaks in period 3 as evidenced by the higher switching cost. However, the marginal gain of waiting until this peak occurs is offset by the increased cost, the concavity of the demand function, and the effect of discounting. This is significant because it highlights the fact that the intuitively appealing idea of waiting until the cost gap is maximized before switching technologies if only one switch is to be considered is indeed sub-optimal in this setting. Note that this is true even though the slope of the switching cost is relatively low, ($s = 1$).

3. Generalization to multiple adoption case

The single adoption case may be relevant to a small firm with severely limited investment capital. However, the more general case must allow the firm to upgrade its technological asset base whenever it is profitable to do so. Thus, we extend the model of Section 2 to consider an infinite horizon allowing multiple adoption points.

3.1. Revenue and switching cost

The maximized net revenue for a firm operating in period t with a technology of age n has already been stated. This form does not change as we move to a multiple adoption setting, thus $\Pi(n, t) = (M - C\alpha^n\rho^z)^2/4\sigma$. (Recall, that z is simply $t - n$.)

At time t, the lowest possible production cost is $C\rho^t$. If the latest adoption took place in period z, the current cost is $C\alpha^n\rho^z$. Therefore the switching cost can be written as, $K(n,t) = k + sC\rho^z(\alpha^n - \rho^n)$. Thus both revenue and switching costs are fully described by the two parameters, n and t.

3.2. DP formulation of adoption decision

A rational decision maker is faced with the following options. He can continue using the technology of age n that is currently in place or he can adopt a new technology introduced in the current period t. In either case he will select the profit maximizing output rate and realize a net profit as indicated in Eq. (5). He then enters period $(t+1)$ with a technology of age $(n+1)$ or of age one depending upon his last decision, and proceeds optimally from that point forward. If we define the present value of all profit maximizing present and future actions as $G(n,t)$, we may state the decision problem as an effort to realize $G(n,t)$ defined as;

$$G(n,t) = \max\left\{\begin{array}{l}\Pi(n,t) + \beta G[n+1, t+1], \\ \Pi(0,t) - K(n,t) + \beta G[1, t+1]\end{array}\right\} \tag{8}$$

where $\Pi(n,t)$ is the maximized profit using a technology of age n for period t. $G[n+1,t+1]$ is the result of optimal behavior looking forward from period $(t+1)$ if we enter period $(t+1)$ with a technology of age $n+1$. $G[1,t+1]$ has a parallel definition.

3.3. Solution procedure

Although (8) presents an infinite horizon problem, we can deduce a finite horizon (V) and a payment vector for this final period for all possible states (n,V) that reflects all profit maximizing actions from that point forward. Consider two firms. The first (firm A) uses the technology that was in place at time $t=0$, with an initial marginal cost C, and never switches technologies. The second (firm B) uses the best available technology in each period but pays no switching cost. We have already stated the maximum profit for firm A in a single period in Eq. (5). Since firm A never changes technology, it earns a similar profit in each period. The production cost changes as a result of learning and the profit maximizing output rate rises in a predictable way. Consequently, we may state the present value (at time 0) of firm A's net revenues over an infinite horizon as, $A_0 = \sum_{n=0}^{\infty}(1/4\sigma)(M - C\alpha^n)^2\beta^n$. We recall that $\sum_{n=0}^{\infty}x^n = 1/(1-x)$ as long as $0 < x < 1$. Thus we may expand this to yield;

$$A_0 = \frac{M^2}{4\sigma(1-\beta)} - \frac{MC}{2\sigma(1-\alpha\beta)} + \frac{C^2}{4\sigma(1-\alpha^2\beta)} \tag{9}$$

Similarly, we may look forward beginning at any point in time t to state the present value (at time t) of all future revenues for firm A as follows;

$$A_t = \frac{M^2}{4\sigma(1-\beta)} - \frac{MC\alpha^t}{2\sigma(1-\alpha\beta)} + \frac{C^2\alpha^{2t}}{4\sigma(1-\alpha^2\beta)} \tag{10}$$

A similar analysis of the profits for Firm B enables us to define B_t as the present value (at time t) of all future revenues accruing to firm B over an infinite horizon beginning at period t.

$$B_t = \frac{M^2}{4\sigma(1-\beta)} - \frac{MC\rho^t}{2\sigma(1-\rho\beta)} + \frac{C^2\rho^{2t}}{4\sigma(1-\rho^2\beta)} \tag{11}$$

Recall that k is the fixed portion of the switching cost and thus its lower bound. Finally, we define $H_t = B_t - A_t$, and V as the smallest integer greater than $\ln[2\sigma k(1-\alpha\beta)/MC]/\ln[\alpha]$ and introduce Proposition 3.

Proposition 3. *It is never optimal to switch in any period $t \geqslant V$.*

This result follows from the facts that H_t is an upper bound on the benefit of switching to a new technology, and for t values greater than V, H_t is less than k and monotonically decreasing. [3]

Comparative statics reveal that V rises with the product's initial production cost or market size (C, or M) and the discount factor β. V also rises when learning is less significant (higher α). A shorter horizon is relevant if the fixed portion of the switching cost rises (k) or if the slope of the demand curve is higher (σ). For the problem setting defined by the parameter values established in Section 2.3, $\ln[2\sigma k\ (1-\alpha\beta)/MC]/\ln[\alpha] = 68.17$, thus it is never optimal to switch after period 68.

The significance of Proposition 3 is that when looking forward from period V we know that future optimal behavior will not involve switching and that the present value of all future net revenues, $G(n, V)$ is defined by Eq. (12) below.

$$G[n,V] = \frac{M^2}{4\sigma(1-\beta)} - \frac{MC\rho^{(V-n)}\alpha^n}{2\sigma(1-\alpha\beta)} + \frac{C^2\rho^{2(V-n)}\alpha^{2n}}{4\sigma(1-\alpha^2\beta)} \tag{12}$$

The payoff vector for period V can be calculated for all n values using Eq. (12). Thus standard DP results are applied to generate payoff values and optimal decisions for all earlier periods. Note that the recursion (8) defines a deterministic shortest-path problem through a network of nodes (n, t).

3.4. Structural results

We now deduce the form of the optimal policy. We show that for each period t, there exists a number $n^*(t)$ such that the firm should switch at time t if $n > n^*$ and continue with the existing technology if $n < n^*$. This follows directly from Proposition 4 below.

Proposition 4. *For sufficiently large values of M, $G(0,t) - G(n,t) - K(n,t)$ is non-decreasing in n.*

In other words the net benefit from switching to a new technology, accounting for future optimal behavior and the current switching costs is non-decreasing in n. Consequently, if switching is profitable for any n value at time t, it will also be profitable for any higher n value.

For any period t, the smallest n value for which $G(0,t) - G(n,t) - K(n,t)$ is positive is n^*. Obviously, $n^* \leqslant t$ and when $t = V, n^* = t$. Therefore, if there exist a time t such that $n^* < t$, the following result must be true.

Proposition 5. *Given a technology adopted in period z, there exist values $t_l(z)$ and $t^u(z)$ such that it is optimal to switch iff $t_l(z) \leqslant t \leqslant t^u(z)$; and $t_l(z)$ is bounded from below by an increasing function of z while $t^u(z)$ is bounded from above by a decreasing function of z.*

This defines the optimal policy in the following manner. If the firm enters period t with a technology adopted in period z and $t_l(z) \leqslant t \leqslant t^u(z)$ then it is optimal for the firm to adopt a new technology immediately. Otherwise, the firm should continue with its present technological base. This result follows from Propositions 3 and 4 and the form of the switching cost. It implies that if a technology is adopted in period z, there is a window of time during which we should switch to a new technology. Before we enter this

[3] Note that H_t is increasing for small values of t. An alternative approach is to consider a single switch at time V allowing firm A to move to a production cost of 0. When H_t is both $< k$ and decreasing in t then the Proposition is proven. This approach is computationally simple but may yield a much larger value of V, making the solution unnecessarily lengthy.

window, the switching cost can not yet be justified. If for some reason we were to hold onto a technology until we exit this window then we will keep that technology indefinitely.

3.5. An illustrative example

Let us assume that the same parameter values applied in Section 2.3 are in place and that the firm adopted a new technology at time $t = 0$. Solving the DP shows that it is optimal for such a firm to switch technologies in periods 1–6, 8, 10, 12, 15, and 20. As t increases, adoption occurs less frequently. After period 6 the value of n that motivates switching is increasing in t at a rate faster than t itself. Consequently, an n value that motivates the firm to switch when t is small is not large enough to motivate switching as t rises. Data from the solution of (8) are displayed in Fig. 3.

Here, the shaded area consists of horizontal bars that indicate the combinations of z and t values that motivate a switch. For example, at $z = 18$, we see a shaded area spanning from $t = 23$ to $t = 29$. Thus the optimal policy is to switch technologies if the last adoption point was period 18 and the current time is between 23 and 29. The solid line shows the behavior of the firm in our example. Whenever the line is touching the shaded area, the firm will switch to a new technology. Thus, there is an upward jump at periods 1–6, 8, 10, 12, 15, and 20. For example, we see that in period 15 the solid line meets the shaded area indicating that the firm will adopt a new technology, thus the line jumps to the value of 15. This value is optimal until period 20, at which time the firm again adopts a new technology. This is indicated by the final "jump" which results in the firm permanently leaving the region of adoption.

If the firm, for some reason, were to keep its technology adopted in period 15 until period 35, then switching would no longer be optimal. This is an illustration of Proposition 5. This result is meaningful because it more accurately describes a number of settings that seem to contradict intuition. For example,

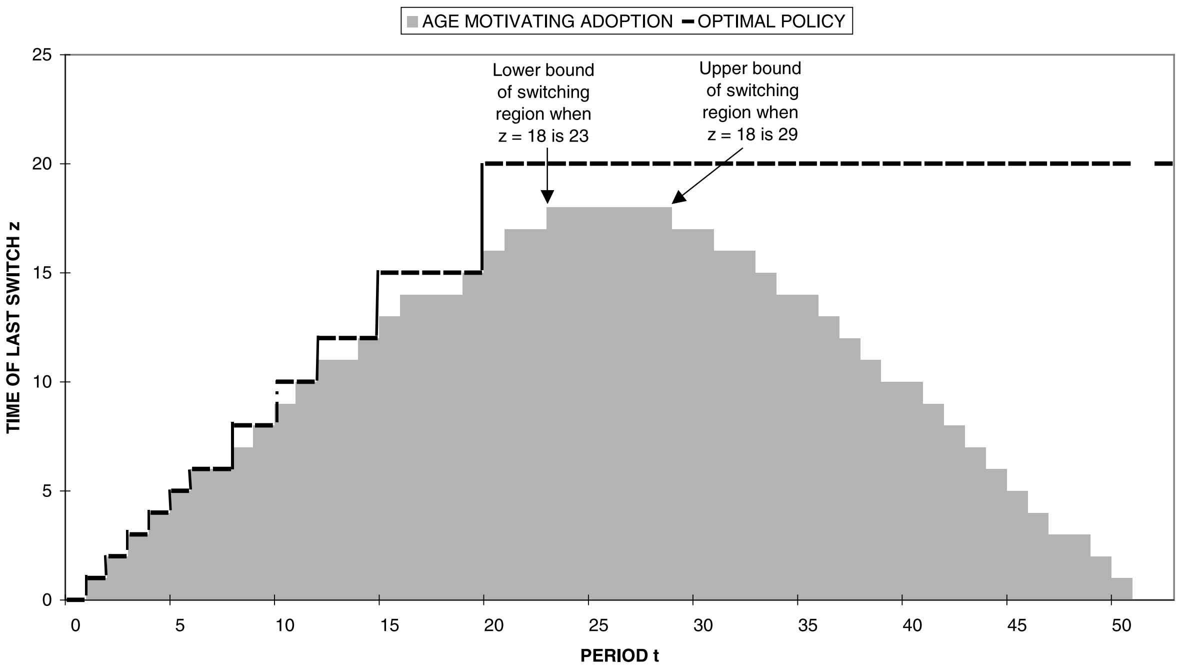

Fig. 3. Optimal policy and firm behavior for base case.

NASA and much of the US Defense Department continues to utilize millions of lines of computer code written in FORTRAN even though it is commonly agreed that more recently designed languages are much more efficient. The reason for this anomaly appears to be that for many years these agencies were hesitant to adopt a new language because the result of programming errors can be so disastrous. An extensive body of work with a language must be in place to demonstrate its robustness before it can be adopted for such sensitive uses. While this body of work is being formed, the users of FORTRAN continued along a learning curve. By the time that a conversion could be considered, literally hundreds of millions of lines of code were in place and being used on a regular basis. Also, thousands of experienced programmers with high security clearances were in place. By this point, the cost of switching to a new language was enormous. Also, the more sophisticated the languages became, the more they differed from the original programming style and the more difficult converting old code to the new languages became. Thus the switching cost was both large and increasing over time. Consequently, even though the technology gap (n) is quite large, the optimal policy today is to continue with the older technology.

3.6. Numerical results

A number of additional insights are suggested by an analysis of problems similar to the example of Section 3.5, hereafter referred to as the "base case". Specifically, if we assume that $0.7 < \rho < \alpha < 1$, then we are able to produce evidence supporting several additional observations, including the following.

Observation 1. *Total profit appears to be more sensitive to changes in ρ than to changes in α.*

Support for this notion is drawn from Figs. 4 and 5. These show profit levels that accrue to a firm under two different policies. In both cases the upper line shows the NPV (at time 0) of implementing the optimal policy, as defined by the solution to the resulting dynamic program. The solid line in Fig. 5 shows this value

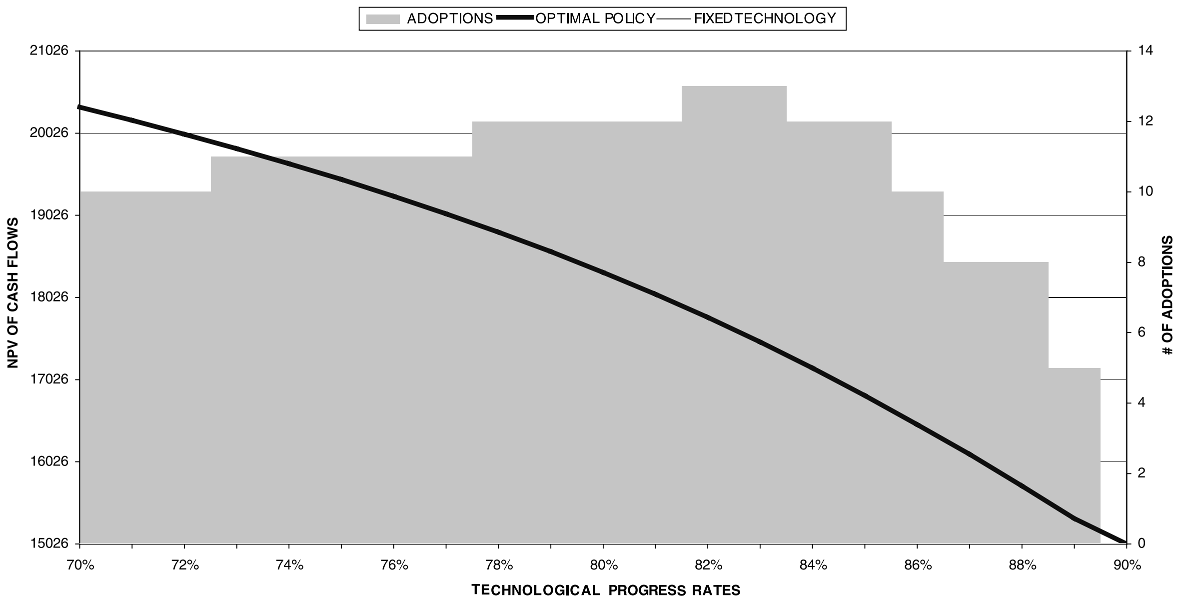

Fig. 4. Profit vs. progress rates.

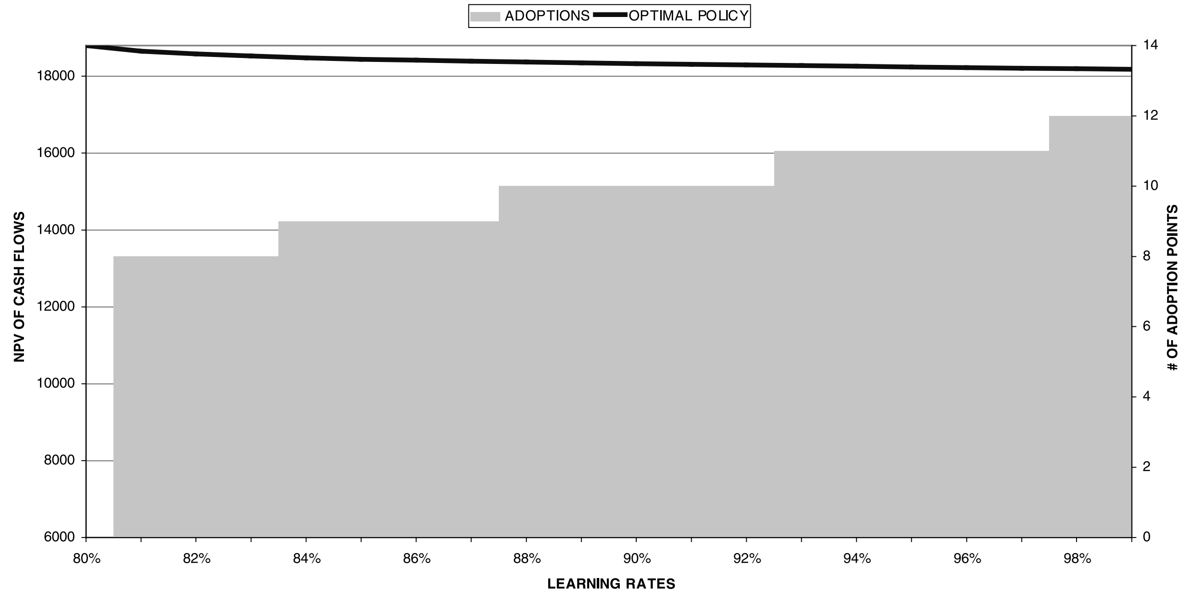

Fig. 5. Profit vs. learning rates.

for a fixed learning rate ($\alpha = 90\%$) with varying levels of technological progress ($70\% \leqslant \rho \leqslant 90\%$). These values may be compared to the policy of treating the technological asset base as "fixed". By fixed we mean that learning takes place but that no new technology is ever adopted. In these cases profit is unaffected by changes in ρ. Specifically, producing with $\alpha = 90\%$ and never switching technologies produces a payoff of 15026 for all ρ values and is represented by the horizontal line that doubles as the x axis. The shaded region of Fig. 4 is constituted of vertical bars showing the number of adoption opportunities that the firm will take advantage of under the optimal policy. The scale for these bars is shown on the secondary vertical axis on the right hand side (RHS) of the figure.

Fig. 4 is in complete agreement with the intuition that when the optimal policy is applied, greater rates of technological progress (lower ρ values) produces greater profits and the gap between the payoffs of the optimal and fixed policies is increasing in a concave way.

By contrast Fig. 5 presents payoff values for cases where ρ is fixed (in this case at 80%) and α values range from 80% to 100%. The solid line indicates the payoff from the optimal policy while the dashed line indicates the payoff from the fixed technology case. What is most dramatic about this figure is that the slope of the line indicating the NPV of the optimal policy is very close to 0. Thus Observation 1 is simply a restatement of the fact that the corresponding curves in Figs. 4 and 5 show dramatically different slopes.

Observation 2. *The optimal policy is not sensitive to changes in the discount rate.*

This is not meant to imply that profit is not dramatically affected by β values; only that the optimal behavior, in terms of when the firm switches to a new technology is not significantly affected by changes in discount rates. Consider Fig. 6. Here we note that increases in β values dramatically increase the NPV of any reasonable policy. However, if we focus upon the optimal number of adoption points, we see that this number is completely unaffected by changes in β. Since both the benefits and the costs of switching to a new technology are discounted in exactly the same way, discounting has a dramatic effect on the present value of net profit but very little effect on the optimal behavior.

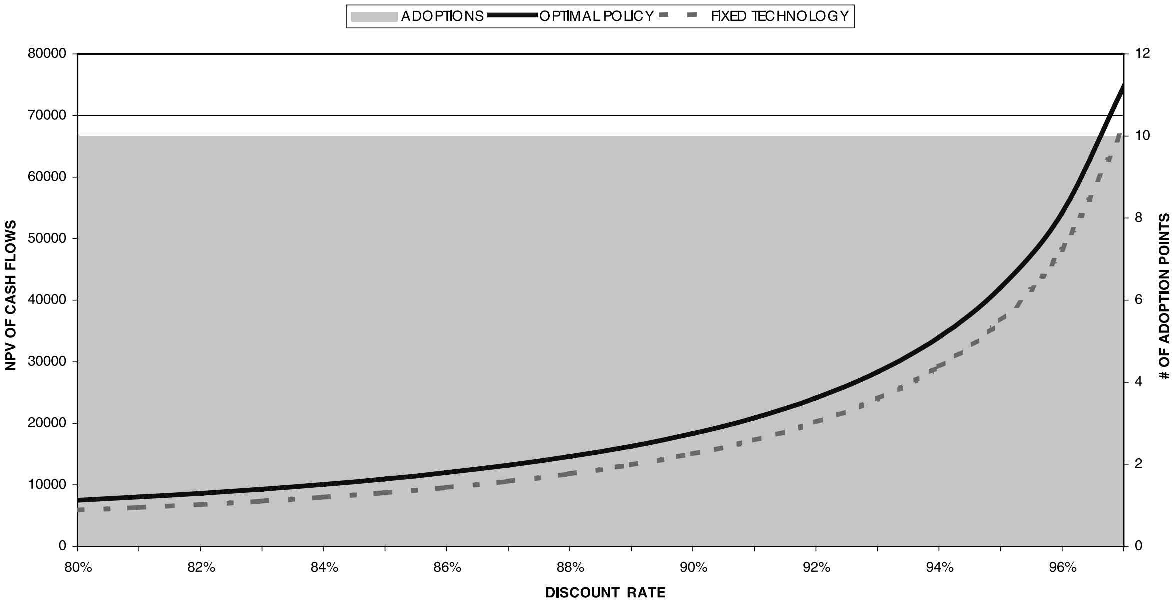

Fig. 6. Profit vs. beta.

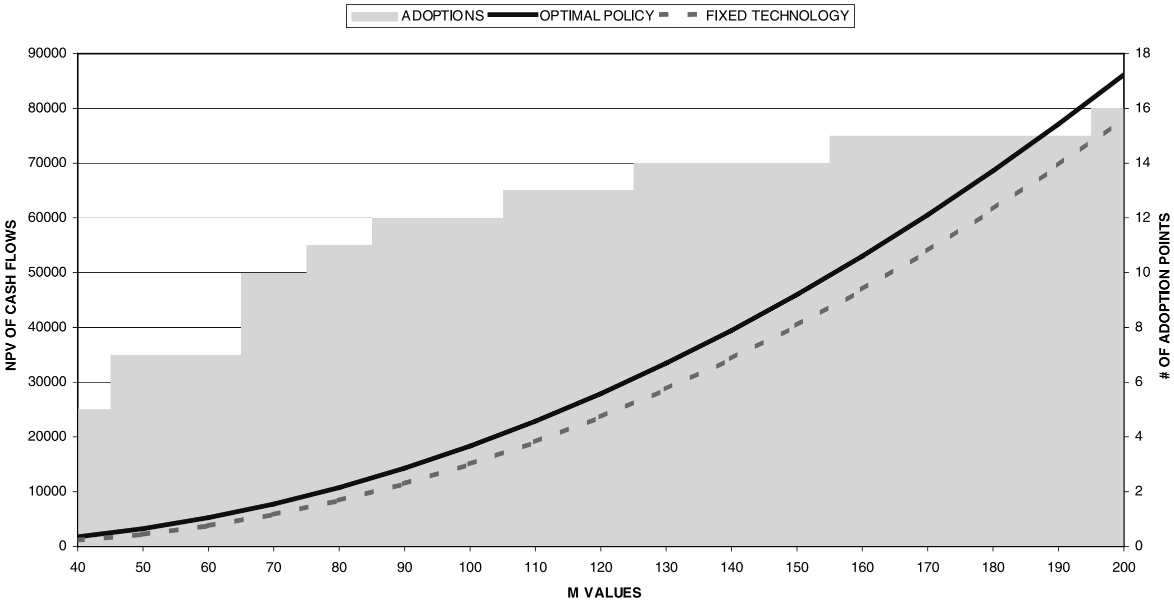

Fig. 7. Profit vs. market size.

Observation 3. *The number of adoption points is increasing in M, C, and α, and decreasing in k and s.*

This observation is drawn from Fig. 5 (for ρ), Fig. 7 (for M), Fig. 8 (for C), Fig. 9 (for k), and Fig. 10 (for s). These facts are generally not surprising. Larger markets or higher initial cost make it easier to justify

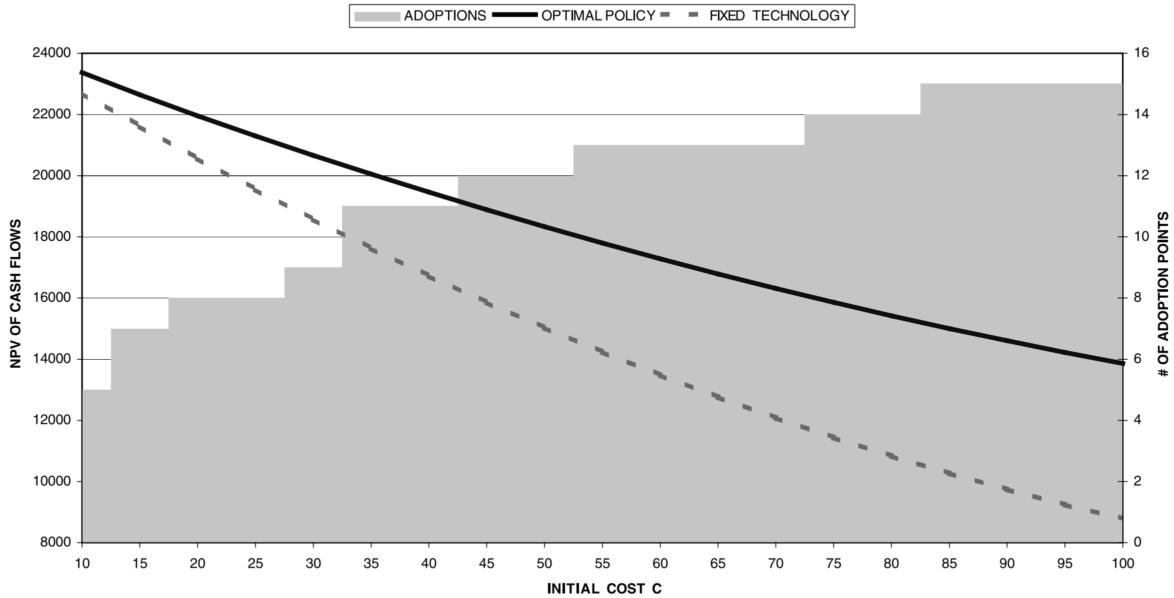

Fig. 8. Profit vs. initial cost.

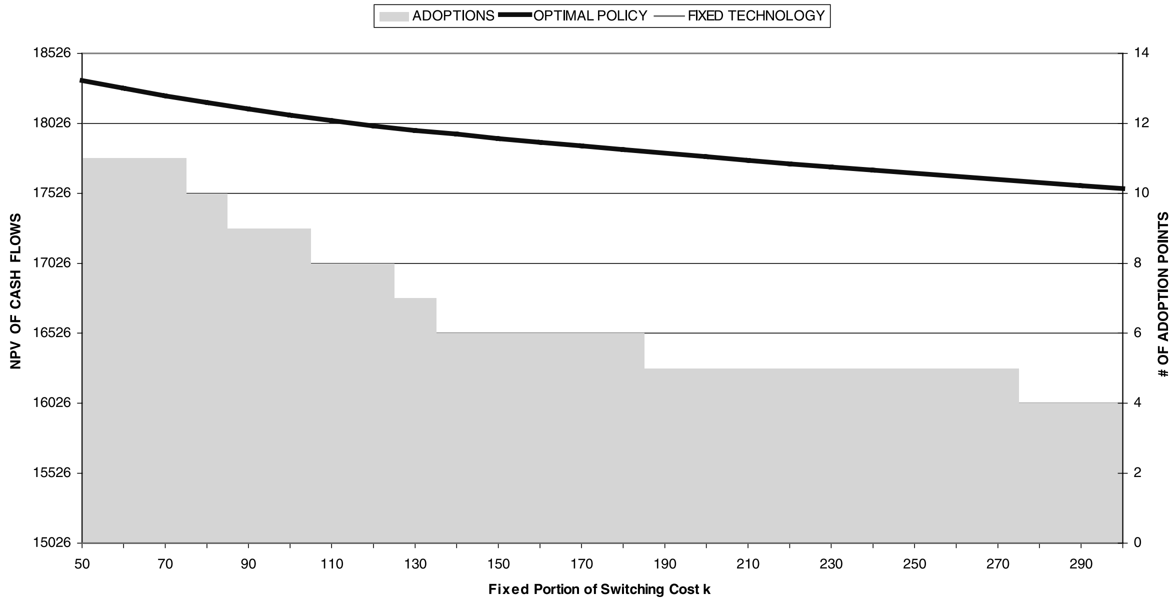

Fig. 9. Profit vs. fixed portion of switching cost.

investments in cost reduction, and increasing the acquisition price of cost reducing technologies will make them less attractive. What is most significant here is the insight into how behavior changes with ρ values.

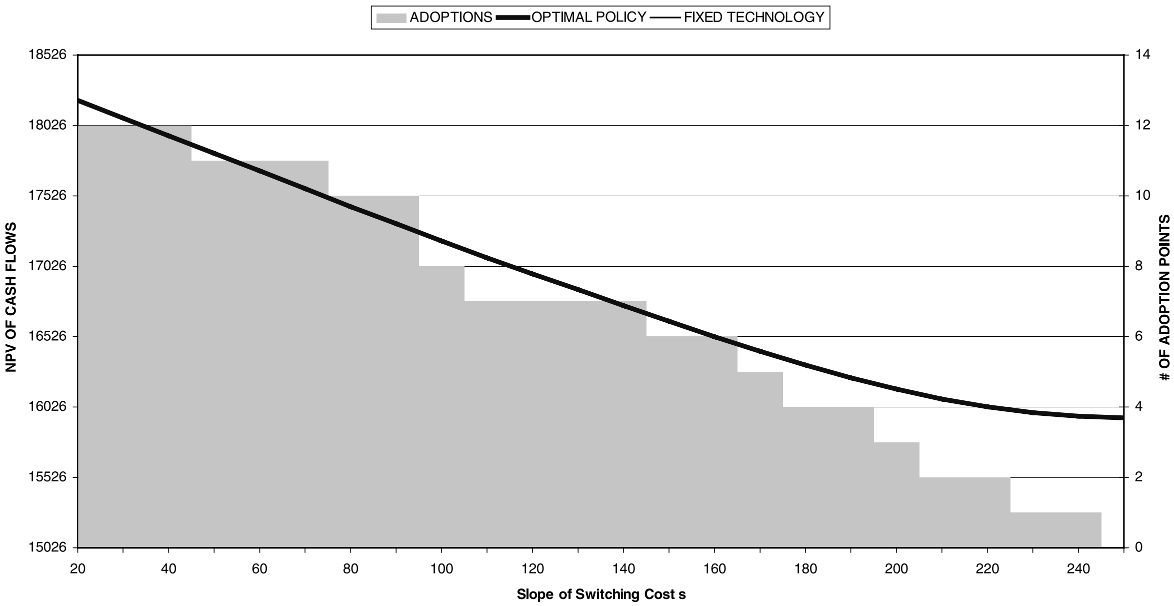

Fig. 10. Profit vs. slope of switching cost.

Table 2
Number of adoption points

α Values	ρ Values				
	0.7	0.75	0.80	0.85	0.90
0.75	5				
0.80	7	6			
0.85	9	9	8		
0.90	10	11	12	11	
0.95	11	13	15	16	17

Recall from Proposition 2 that adoption tends to be accelerated as ρ rises in the case where only one such move is allowed. However, in this more general case, accelerated technological progress may or may not result in more instances of technology adoption.

Table 2 shows the number of technology adoption points over an infinite horizon for various levels of α and ρ. If we look down any column we see that rising α values always result in an increase in the number of adoption points. However, if we look across any row of this table we see that the pattern is not so simple when ρ values rise.

This more complex relationship involving α, ρ, and the number of optimal adoption points was also shown in Fig. 4 where the number of adoption points rises with ρ between 70% and 83% and then declines for higher values. The nature of this interaction stems from the fact that adoption decisions are not driven singularly by ρ, but by the interaction among several parameters. The number of adoption points does depend upon the gap between α and ρ. But it also depends upon how long this gap dominates the scenario. This duration is not a simple function of ρ.

4. Conclusions

Taking a dynamic programming approach, we have analyzed the optimal timing of technology adoption by a firm experiencing learning at a deterministic rate as well as technological progress resulting from efforts external to the firm. Focusing on the benefits and costs resulting from decisions to switch technologies illustrates that the technology gap motivating switching is not fully described by the time since the last adoption "n", nor is it simply described by the gap between learning and progress rates. In fact the interplay among learning, technological advancement, and discounting leads to several less than obvious results. First, increased learning rates or decreased technological progress rates may actually stimulate early adoption when only one adoption is considered. Second, a large or growing market or a steeply sloping demand curve may serve to delay technology adoption. Third, when multiple adoption points are allowed, increasing the time since the last adoption may move the decision maker from a range where switching is optimal to one in which this is no longer the case. Finally, higher rates of technological progress may serve either to increase or decrease the number of adoptions that the firm will make.

Appendix A

Proposition 6. *Under Assumptions A and B, Φ is uni-modal.*

Proof. This is equivalent to stating that if Φ is decreasing in t over a range from t to $(t+1)$ then it cannot be increasing in t over a range from $(t+1)$ to $(t+2)$. We restate our assumptions here as: [A] $C > 4\sigma/(1-\alpha^2)\alpha^2$, and [$B$] $M > 2C\{\alpha(1-\alpha)^2 + (1-\rho)^2\}/(1-\alpha)(1-\rho^2)$. Now, assume: $\Phi(t+1) > \Phi(t)$. This implies that,

$$\frac{M^2}{4\sigma} + \left(\frac{1-\rho^2\beta}{1-\alpha^2\beta}\frac{C^2\rho^{2t}}{4\sigma}\right) + \beta K(t+1) - \beta\Pi(t+1) - K(t) - \frac{1-\rho\beta}{1-\alpha\beta}\frac{2MC\rho^t}{4\sigma} \geqslant 0 \tag{A.1}$$

Rearranging terms yields,

$$\left(\frac{1-\rho^2\beta}{1-\alpha^2\beta}\frac{C^2\rho^{2t}}{4\sigma}\right) \geqslant \beta\Pi(t+1) + \frac{1-\rho\beta}{1-\alpha\beta}\frac{2MC\rho^t}{4\sigma} + K(t) - \frac{M^2}{4\sigma} - \beta K(t+1) \tag{A.2}$$

We wish to show that, Φ must also be increasing from $t+1$ to $t+2$. In other words that,

$$\frac{M^2}{4\sigma} \geqslant \beta\Pi(t+2) - \beta K(t+2) + K(t+1) + \frac{1-\rho\beta}{1-\alpha\beta}\frac{2MC\rho^t}{4\sigma}\rho - \left(\frac{1-\rho^2\beta}{1-\alpha^2\beta}\frac{C^2\rho^{2t}}{4\sigma}\right)\rho^2 \tag{A.3}$$

Note that the term on the left hand side of (A.2) appears on the RHS of (A.3). Thus (A.3) is true if we can meet the stronger condition created by substituting the RHS of (A.2) into (A.3) and gathering terms as shown in (A.4).

$$M^2 \geqslant \frac{4\sigma}{1-\rho^2}\left\{\begin{array}{l}\frac{1}{4\sigma}(C^2\alpha^{2t+4} - C^2\alpha^{2t+2} + 2MC\alpha^{t+1} - 2MC\alpha^{t+2}) \\ +\frac{1-\rho\beta}{1-\alpha\beta}\frac{2MC\rho^t}{4\sigma}(1-\rho)\rho + F\end{array}\right\} \tag{A.4}$$

where, F is a residual term guaranteed to be less than 1. Clearly the RHS is maximized when $t = 0$, and $\beta = 1$. Thus we set $t = 0$, $\beta = 1$, $F = 1$, rearrange terms, and verify,

$$M^2 \geqslant \frac{1}{1-\rho^2}\{C^2\alpha^2(\alpha^2-1) + 2MC\alpha(1-\alpha)\} + \frac{1-\rho}{1+\rho}\frac{2MC}{1-\alpha} + \frac{4\sigma}{1-\rho^2} \tag{A.5}$$

We may establish how the RHS of (A.5) is maximized by rearranging terms to yield (A.6).

$$M^2 \geqslant \frac{C^2\alpha^2(\alpha^2-1)}{1-\rho^2} + 2MC\frac{\alpha(1-\alpha)^2+(1-\rho)^2}{(1-\alpha)(1-\rho^2)} + \frac{4\sigma}{1-\rho^2} \tag{A.6}$$

Since $\alpha < 1$, the first term on the RHS must be negative. This term also shares the same denominator as the last term. We see that their sum is negative as long as $C^2 > 4\sigma/(1-\alpha^2)\alpha^2$ as stated in Assumption A. Consequently, the first and last terms may be dropped, thus the proposition is true as long as, $M \geqslant 2C\ (\alpha(1-\alpha)^2+(1-\rho)^2)/((1-\alpha)(1-\rho^2))$ as stated in Assumption B. □

Proposition 7. *T is increasing in k, and M, and decreasing in s. When M is sufficiently large T is also increasing in* β, *and decreasing in* σ.

Proof. Note that Ψ is always linear in M and consider $\partial\Psi/\partial M = ((\rho\beta-1)C\rho^t)/2\sigma(1-\alpha\beta) + C\alpha^t/2\sigma$. Note that this is *negative* until, $(\alpha/\rho) \geqslant ((1-\rho\beta)/(1-\alpha\beta))^{1/t}$. We see that,

$$\frac{\partial\Psi}{\partial C} = (\rho\beta-1)\rho^t\left\{\frac{M}{2\sigma(1-\alpha\beta)} - s\right\} + \alpha^t\left\{\frac{M}{2\sigma} + (\alpha\beta-1)s\right\} + \frac{(1-\beta\rho^2)}{2\sigma(1-\alpha^2\beta)}C\rho^{2t} + \frac{C\alpha^{2t}}{2\sigma} \tag{A.7}$$

(A.7) is negative if,

$$C \leqslant \frac{(1-\alpha^2\beta)\{\rho^t(1-\rho\beta)(M-2\sigma s(1-\alpha\beta)) + \alpha^t(1-\alpha\beta)^2 2\sigma s - \alpha^t M(1-\alpha\beta)\}}{(1-\alpha\beta)\{(1-\rho^2\beta)\rho^{2t} + (1-\alpha^2\beta)\alpha^{2t}\}} \tag{A.8}$$

Unfortunately, (A.8) is not uniformly true or false for the parameter values used here. However, we can say that raising C values tends to postpone adoption if C were low to begin with but accelerates it if C is large.

$$\frac{\partial\Psi}{\partial\alpha} = \frac{C}{2\sigma}\left\{\begin{array}{l}\dfrac{C\alpha\beta\rho^{2t}(1-\rho\beta^2)}{(\alpha^2\beta-1)^2} - \dfrac{M\beta\rho^t(1-\beta\rho)}{(\alpha\beta-1)^2} \\ +\alpha^{t-1}(Mt + Ct\alpha^t + 2s((t+1)\alpha\beta\sigma - t\sigma))\end{array}\right\} \tag{A.9}$$

$$\frac{\partial\Psi}{\partial\rho} = \frac{C\rho^{t-1}}{2\sigma}\left\{\frac{C\rho^t((t+1)\rho^2\beta-t)}{\alpha^2\beta-1} + \frac{\sigma((t+1)\rho\beta-t)(M+2s(\alpha\beta-1))}{1-\alpha\beta}\right\} \tag{A.10}$$

Rearranging terms shows that $\partial\Psi/\partial\alpha$ is negative as long as,

$$M \geqslant \left\{\beta\rho^t\frac{(1-\beta\rho)}{(1-\alpha\beta)^2} - t\alpha^{t-1}\right\}^{-1}\left[\begin{array}{l}C\left\{\alpha\beta\rho^{2t}\dfrac{1-\rho^2\beta}{(1-\alpha^2\beta)^2} + t\alpha^{2t-1}\right\} \\ +s\{2\sigma\alpha^{t-1}(\alpha\beta t + \alpha\beta - 1)\}\end{array}\right] \tag{A.11}$$

Similarly, rearranging terms shows that $\partial\Psi/\partial\rho$ is *positive* as long as,

$$M \geqslant C\frac{1-\alpha\beta}{1-\alpha^2\beta}\,\frac{\rho^t(\beta\rho^2+\beta\rho^2 t - t)}{\sigma(\beta\rho+\beta\rho t - t)} + s2(1-\alpha\beta) \tag{A.12}$$

Thus for sufficiently large M, T is increasing in α and decreasing in ρ.

Values for $\partial\Psi/\partial\beta$ must be negative as,

$$M \geqslant \frac{2\sigma(1-\alpha\beta)}{\rho^t}\left\{\frac{\alpha(1-\beta\rho)}{1-\alpha\beta} - \rho\right\}^{-1}\left[\begin{array}{l}C\rho^{2t}\left\{\dfrac{\alpha^2(1-\rho^2\beta)-\rho^2}{4\sigma(1-\alpha^2\beta)}\right\} \\ +\alpha(\alpha^{t+1}-\rho^{t+1}) + k/C\end{array}\right] \tag{A.13}$$

Similarly $\partial\Psi/\partial\sigma$ is positive as long as,

$$M \geqslant \frac{C}{2}\left\{\alpha^{2t} + \frac{\rho^{2t}}{4\sigma^2}\frac{1-\beta\rho^2}{1-\beta\alpha^2}\right\}\frac{1-\alpha\beta}{\rho^t(1-\rho\beta)-\alpha^t(1-\alpha\beta)} \tag{A.14}$$

Therefore, T is increasing in β, and decreasing in σ.

Since $\partial\Psi/\partial k = \beta - 1$, and $\beta < 1, \partial\Psi/\partial k$ is always negative, thus T is always increasing in k.

Finally, $\mathrm{d}\Psi/\mathrm{d}s = (1-\beta\rho)\rho^t + (\alpha\beta - 1)\alpha^t$. Thus $\mathrm{d}\Psi/\mathrm{d}s$ is positive until, $((1-\beta\rho)/(1-\beta\alpha))^{1/t} < \alpha/\rho$. □

Proposition 8. *It is never optimal to switch in any period $t \geqslant V$.*

Proof. The proposition is logically equivalent to two statements. (1) $H_V \leqslant k$, and (2) H_t is non-increasing (in t) for all $t \geqslant V$. H_t can be rewritten as, $H_t = W\rho^{2t} - X\rho^t - Y\alpha^{2t} + Z\alpha^t$, where $W = C^2/4\sigma(1-\rho^2\beta)$, $X = MC/2\sigma(1-\rho\beta)$, $Y = C^2/4\sigma(1-\alpha^2\beta)$, and $Z = MC/2\sigma(1-\alpha\beta)$. Clearly since ρ and α are less than 1, H_t goes to 0 as t goes to infinity. However, this does not necessarily mean that H_t is monotonically decreasing for t all values above V. In other words we must show that, $H_t - H_{t+1}$ is non-negative for "large" t values, where large simply means greater than or equal to V. Stated formally, we must verify that,

$$W\rho^{2t}(1-\rho^2) - X\rho^t(1-\rho) - Y\alpha^{2t}(1-\alpha^2) + Z\alpha^t(1-\alpha) > 0 \tag{A.15}$$

Since $\rho < \alpha$, (A.15) will be true if the stronger statement, (A.16) is true.

$$X\rho^t - W\rho^{2t}(1+\rho) < Z\alpha^t - Y\alpha^{2t}(1+\alpha) \tag{A.16}$$

Dividing both sides by $X\rho^t$, rearranging terms and simplifying yields, $1 < (\alpha/\rho)^t\{(Z - Y(1+\alpha)\alpha^t)/X\} + (W/X)\rho^t(1+\rho)$. Clearly, the first term on the RHS is monotonically increasing in t since $\alpha > \rho$. The final term is always positive but approaches 0 as t rises, therefore we focus on the middle term (in braces "{ }"). Clearly Z/X is independent of t, and the negative portion of this term must be decreasing in t since $\alpha < 1$. Therefore when t is "sufficiently large" this inequality must hold and statement (2) must be true.

We may analyze H_t to define "sufficiently large." Inspection shows that $Y > W$. Using this along with the fact that $\alpha > \rho$ ensures that $W\rho^{2t} - Y\alpha^{2t}$ is negative. Thus if $Z\alpha^t - X\rho^t < k$, then statement (1) is also true. This implies that $\{1 - X/Z(\rho/\alpha)^t\} < k/Z\alpha^{-t}$. We may drop the negative term on the left hand side noting that it is always between 0 and 1 and establish a simple rule for calculating V by looking at $k/Z\alpha^{-t} > 1$. Rearranging terms shows that V is the largest integer less than $\ln[k/Z]/\ln[\alpha]$. □

Proposition 9. *For sufficiently large values of M, $G(0,t) - G(n,t) - K(n,t)$ is non-decreasing in n.*

Proof. Since the cost gap between a technology introduced in period n and a new one is monotonically increasing in n we know that $\Pi(n,t)$ is decreasing in n and $K(n,t)$ is increasing in n. By induction we will show that $G(n,t)$ is non-increasing in n. Analysis shows that the Proposition is true if $t = V$ as long as, $(1-\alpha\beta)/(1-\alpha^2\beta)\alpha^n\rho^{V-n} \leqslant (M/C)$. This is true because the LHS is less than one and $M > C$.

Consider period $t-1$. From Eq. (8) we know that,

$$G(n-1,t-1) = \max\left\{\begin{array}{l}\Pi(0,t-1) + \beta G(n,t),\\ \Pi(0,t) + \beta G(1,t)\end{array}\right\}.$$

Thus $G(n-1,t-1)$ must be non-increasing in n whenever this is true for $G(n,t)$.

In any period t, the firm will either switch or it will not. If it does not then $G(0,t-1) = \Pi(0,t-1) + \beta G(1,t)$ and $G(n-1,t-1) = \Pi(0,t-1) + \beta G(1,t) - K(n-1,t-1)$, thus the proposition simply states that;

$$\Pi(0,t-1) + \beta G(1,t) - \Pi(0,t-1) - \beta G(1,t) + K(n-1,t-1) - K(n-1,t-1) \tag{A.17}$$

is non-decreasing in n, however it is clear that (A.17) equals 0. On the other hand if the firm does not switch, then the proposition states that,

$$\Pi(0,t-1)+\beta G(1,t)-\Pi(n-1,t-1)-\beta G(n,t)-K(n-1,t-1) \tag{A.18}$$

is increasing in n. In other words this value is increasing as we move from n to $n+1$. Thus the proposition is proven if,

$$\begin{aligned}&\Pi(0,t-1)-\Pi(n,t-1)+\beta G(1,t)-\beta G(n+1,t)-K(n,t-1)\\&\quad>\Pi(0,t-1)-\Pi(n-1,t-1)+\beta G(1,t)-\beta G(n,t)-K(n-1,t-1)\end{aligned} \tag{A.19}$$

Rearranging and simplifying implies that the proposition is true if,

$$\{K(n-1,t-1)-K(n,t-1)\}+\{\Pi(n-1,t-1)-\Pi(n,t-1)\}>\beta\{G(n+1,t)-G(n,t)\} \tag{A.20}$$

Since $G(n,t)$ is decreasing in n, the RHS is negative. Finally, the statement is true whenever, the LHS is positive. In other words when,

$$\left(\frac{M-C\alpha^{n-1}\rho^{t-n-1}}{2\sigma}\right)^2-\left(\frac{M-C\alpha^{n}\rho^{t-n}}{2\sigma}\right)^2\geqslant\left(\frac{\alpha}{\rho}-1\right)sC\rho^t\frac{\alpha^{n-1}}{\rho^{n-1}} \tag{A.21}$$

Simplifying and rearranging terms yields,

$$M\geqslant\frac{(\alpha^2-\rho^2)C\alpha^{n-1}\rho^{t-n-1}}{2\left(\dfrac{\alpha}{\rho}-1\right)}+2s\sigma^2 \tag{A.22}$$

For the problems shown as examples in this work, the first term on the right hand side is less than one, thus (A30) is true whenever $M>(1+2s\sigma^2)$. □

Proposition 10. *Given a technology adopted in period z, there exist values $t_l(z)$ and $t^u(z)$ such that it is optimal to switch iff $t_l(z)\leqslant t\leqslant t^u(z)$; and $t_l(z)$ is bounded from below by an increasing function of z while $t^u(z)$ is bounded from above by a decreasing function of z.*

Proof. Clearly, $t_l(z)\geqslant(z+1)$, and $t^u(V-1)$. Since $(z+1)$ is an increasing function of z, this part of the proposition is obvious.

Define the variable manufacturing cost per unit immediately after adopting a new technology in period z as $C(z)=C\rho^z$. Since $K=k+sC\rho^z(\alpha^n-\rho^n)$ there is no reason to switch at time t simply to make it easier to switch at any time later than t. Therefore the firm never adopts a new technology unless the gain in net profit accruing between time z and the time of the next adoption exceeds the switching cost. The gain in net profit until the next switch is bounded from above by a function that must be proportional to $C(z)$ $\{\alpha^n-\rho^n\}$. While this function is initially non-decreasing, and later non-increasing in n, we add that it is strictly decreasing in z because $C\rho^z$ is strictly decreasing in z since $(\rho<1)$. Consequently, the upper limit on the range in which the present value of the net revenue gain from switching is greater than the switching cost must drop as z rises. □

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